

The Fate of the Initial State Fluctuations in Heavy Ion Collisions. II Glauber Fluctuations and Sounds

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Heavy ion collisions at RHIC are well described by the (nearly ideal) hydrodynamics for average events. In the present paper we study initial state fluctuations appearing on event-by-event basis, and the propagation of perturbations induced by them. We found that (i) fluctuations of several lowest harmonics have comparable magnitudes, (ii) that at least all odd harmonics are correlated in phase, (iii) thus indicating the local nature of fluctuations. We argue that such local perturbation should be the source of the “Tiny Bang”, a pulse of sound propagating from it. We identify its two fundamental scales as (i) the “sound horizon” (analogous to the absolute ruler in cosmic microwave background and galaxy distribution) and (ii) the “viscous horizon”, separating damped and undamped harmonics. We then qualitatively describe how one can determine them from the data, and thus determine two fundamental parameters of the matter, the (average) *speed of sound* and *viscosity*.

PACS numbers:

I. INTRODUCTION

Starting the introduction, let us note that the issues to be discussed in this paper are somewhat similar in nature to current trends in cosmology of the last decade. While the very existence of cosmic microwave background radiation had dramatically confirmed the existence of the Big Bang already in the 1960’s, it is its more recent observations which made cosmology a really quantitative science. Small temperature fluctuations on top of the overall Hubble expansion have been seen on the sky. Their angular size and the magnitude of various harmonics tell us, in surprising detail, about sounds propagating the Universe at the plasma neutralization time, providing precise timing of the cosmological expansion.

Experimental data obtained in heavy ion collisions at the Relativistic Heavy Ion Collider (RHIC) has found the “Little Bang”, a hydrodynamical explosion driven mostly by pressure of the new form of matter, Quark-Gluon Plasma. Their experimental data for radial and elliptic flows has been compiled in the so called “white papers” of all the experiments in 2004, and compared with predictions of relativistic hydrodynamics. Very recent results from the Large Hadron Collider (LHC) on elliptic flow [1] also turned out to be in agreement with hydrodynamical predictions, suggesting that QGP remains a good liquid even at LHC (see e.g. [2]). (For clarity, we mean hydrodynamics complemented by the hadronic cascade for a correct account of the hadronic stage, see [3–5]. Models which do not do that and use rather arbitrary freezeout don’t get the energy dependence right, neither for RHIC nor for LHC.) Dissipative effects from the QGP viscosity provide only small corrections at the few percent level, see [6–8]. So, by now, we have a good quantitative description of the “Little Bang”.

This paper is however not about it, but about phenomena which we will call collectively “The Tiny Bang”. In

the hydrodynamical studies mentioned above one ignored any fluctuations in the system, using average smooth initial conditions, possessing the symmetries of the average collisions. (For example, central collisions are azimuthally symmetric, with vanishing nonzero Fourier components of the flow. Non-central collisions were assumed to produce reflection symmetric $\vec{x} \rightarrow -\vec{x}$ distributions at midrapidity, thus only even harmonics are to be nonzero.) Yet local perturbation of those should produce extra excitation of the “Little Bang”, that should not in general respect such symmetries.

Such *perturbations* of the average explosion can come from at least two different sources. The one which we will study in this paper is due to quantum fluctuations in the wave function of the colliding nuclei, which creates “bumpy” distributions of matter, for any collision, which one can decompose into a smooth average one plus local perturbations.

Another one, to be studied in subsequent papers of this series, are created by the energy deposited by jets propagating through the medium. It has been recently dramatically shown by ATLAS collaboration [9] that even jets with energy above 100 GeV deposit large part of their energy, and sometimes all of it, into the medium. The first ideas were to look at the resulting perturbations in the form of a Mach cone [10], driven by the view that the energy is deposited more or less homogeneously along the jet path. However more recent developments of the theory, based on AdS/CFT, have lead to the view that the deposited energy grows as the cube of the distance travelled by the jet, $\Delta E \sim L^3$, with significant deposition near the endpoint. Thus one may think of the *second kind of the “Tiny Bangs”*, this time occurring roughly half time between the beginning and the end of the “Little Bang”.

The smallness of the perturbation amplitude, with respect to the local density of ambient matter, would sug-

gest the appearance of divergent sound waves, see Fig.1. Similar to the circles from a stone thrown into a pond, hydrodynamics tells us that initial perturbations should become moving waves, with basically nothing left at the original location at later time. This is the picture we are going to work on in this paper.

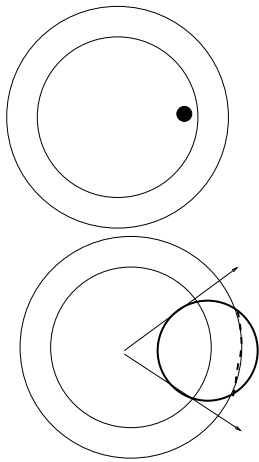


FIG. 1: A sketch of the transverse plane of the colliding system: the two concentric circles are the nuclear radius (inner) and the final radius of the fireball (outer). The black spot in the upper figure is an extra density at formation time, due to initial density fluctuations in the collision. The small perturbation becomes a circle of a sound wave, which will be stopped at freezeout at thick circle (distorted by radial flow). The part outside of the fireball does not exist: the corresponding matter will actually be placed near the edge of the fireball (thick dashed line). The whole perturbation is enclosed in a sector between the two thin lines with arrows.

An alternative idea, of randomly fluctuating shapes of the produced initial fireballs, has resulted in an approach in which different angular harmonics of that distribution are treated separately. The realization that even central $\vec{b} = 0$ collisions may have some fluctuating ellipticity has lead to the discussion of the elliptic flow event-by-event fluctuations, see [11] and many subsequent works. The so called “triangular flow” related to the 3-ed harmonic of the flow has been recently studied by Alver and Roland [12], with several groups working in this direction now.

The main difference between our approach and that is that we treat such fluctuations not as independent noise in different harmonics but as certain local perturbations, resulting in certain evolving sound fronts, reaching certain size, shape and diffusivity by the moment of freeze-out. On one hand, one may argue that as soon as all the perturbations are small and the equations are linear, it is not important if one expands in harmonics before or after the solution of hydro equations. And yet we believe that our approach is not only more intuitive and provides better insights, but it corresponds to the physical nature of the fluctuations in question, as higher angular harmonics are in fact correlated with each other as we show below. Furthermore, we think that our approach is

in more direct connection with the fundamental scales of the problem, to be detailed below.

Having outlined the main ideas of this paper, let us discuss a bit more some recent relevant papers. In the last few years RHIC experiments have focused more on two and three-particle correlations, which revealed a rather rich phenomenology of correlations. We are not however able to review those, and we will refer to data as we develop the theory more.

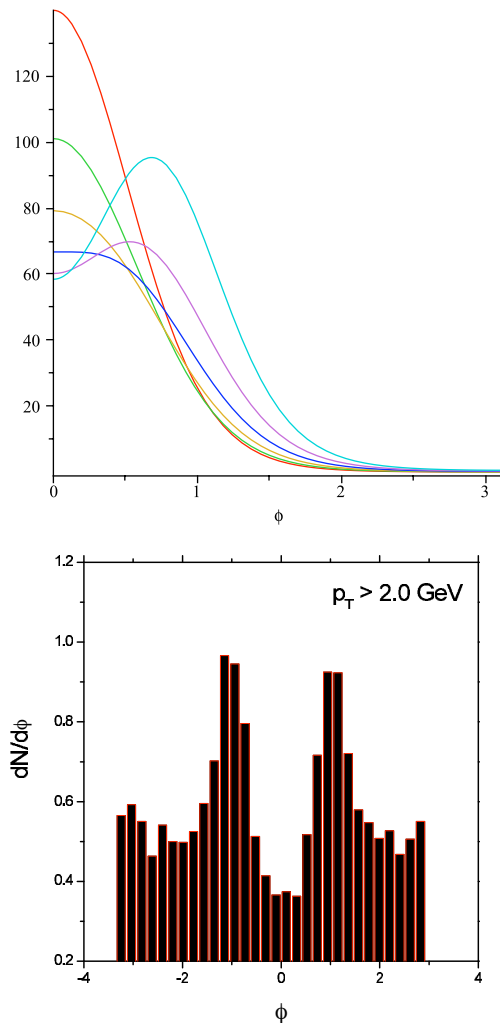


FIG. 2: Extra particle distribution in azimuthal angle relative to position of the initial perturbation. Fig.(a) from [13] has six curves, from the most narrow to wider ones, correspond to the radius of the circle 1,2,3,4,5,6 fm, respectively: the last (blue) with a maximum away from the original position, corresponds to the sound horizon. The original spot position is selected to be at the edge of the nuclei and the observed particle is at $p_t = 1 \text{ GeV}$.

Fig.(b) from [14], for a particle with $p_t > 2 \text{ GeV}$

The propagation of sound on top of the fireball has been discussed by J.Casalderrey-Solana and one of us in [16]. In that paper the fireball expansion was modelled

by the Big-Bang-like overall expansion of the space, with the same Friedman-Robertson-Walker metric as used for cosmology. The focus of that paper was the effect of time-dependent sound velocity, especially if the phase transition is 1st order and it can vanish at some interval of T . The interesting finding was a creation of the secondary – and convergent – sound waves. This idea was further discussed in [13] in connection with the “soft ridge” issue, but with the conclusion that if the current lattice data on the speed of sound is correct, the effect of the reflected wave is too small.

In the same paper [13] it has been found that the usual (unreflected) sound propagation should produce characteristic “two-peak events”, with the angle between the peaks reflecting the sound horizon and numerically being about 1 rad, see the blue curve in Fig. 2(a). Andrade et al [14] have independently come to the same conclusion, see Fig. 2(b) taken from their paper. Note that their peaks are at the same angle: they are more pronounced simply because the particle p_t for which it is plotted is higher. Note also a dip near zero, indicating that nothing remains at the original location of the bump. Andrade et al have further pointed out that the *two*-peak events lead to a *three*-peak correlation function shown in Fig. 3, with the side peaks now at twice larger angle ± 2 rad, now on the “away” side from the peak. (Note that the central peak is about twice larger than the peripheral ones: this is because it corresponds to events 11 and 22 while the other peaks are 12 and 21 combinations, from the peaks 1 and 2 in the two-peak distribution.)

This observation explained what has been found earlier, in the “event-by-event” hydrodynamical studies by the Brazilian group (see [18] and references therein). The Brazilian group has used the initial condition from string-based model developed by Werner and collaborators. Recently this group also studied event-by-event hydrodynamics [15] and claimed that their correlation function describes the data on two-particle correlations. They trace their origin to multiple “bumps” in the initial distribution that lead to the development of what they called “fingers”. There are about $O(10)$ of them per event, each corresponding to rather narrow angles. Presumably the “event-by-event” complicated pictures are more or less linear superpositions of the independent two-peak structures from each bump discussed above, because the sound waves (as *any* Goldstone modes) hardly interact with each other. We will discuss all those issues in detail: and for now let us only note that all these works have *not* yet included the key phenomenon to be studied, namely *viscosity*. It is easy to estimate that even the minimal conjecture viscosity $\eta/s = 1/4\pi$ would significantly modify the amplitudes of those fluctuations.

The most striking outcome of all these works [12, 15, 18] is that they have obtained the “Mach-cone-like” correlation functions, with the away-side ridges at $\pm 2\pi$ (this in the other hemisphere from the trigger) without any jets involved! This created large enthusiasm, and attempts to disregard hard collisions and jets, even for

the events with trigger hadrons of few GeV p_t . However, there is no doubt that such particles would not be produced from hydrodynamics and are still due to hard collisions producing jets.

Although this work is in many respects a continuation of the paper [13], we will focus on sounds and postpone the development of another important idea proposed there. In spite of its phenomenological success, there are good reasons to think that the hydrodynamical description of the RHIC evolution may be incomplete because in the so called M-phase of the collision electric gauge fields should remain unscreened for the time of evolution. It has thus been proposed to upgrade hydrodynamics to (dual)magnetohydrodynamics, including these unscreened electric fields in the stress tensor. This changes the nature of the propagating modes, and it also makes possible the survival of the “flux tubes” in the “QGP corona”. We hope to return to those effects later.

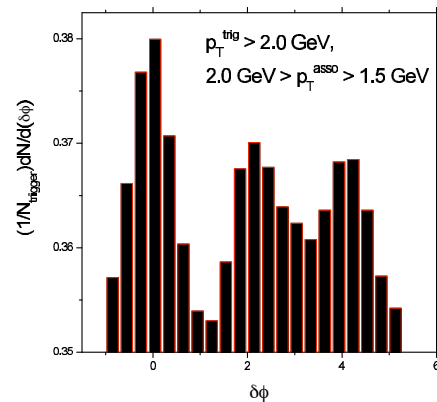


FIG. 3: (from [14]) the two-particle correlation function from the two-peak events, for particles with transverse momenta indicated on the figure.

II. SETTING UP THE PROBLEM

A. The main scales of the problem

Before going to specifics, let us formulate the problem in a more general form, which in a way connects it to the Big Bang fluctuations.

Two generic scales are (i) the macroscopic scale R and the microscopic scale l , being in the relation

$$l \ll R \quad (2.1)$$

which ensures such macroscopic tools as thermo and hydrodynamics to work.

The macroscopic scale R is the size of the fireball in heavy ion collisions and the curvature scale a in the Big bang. Note that both are in principle time dependent,

demonstrating the expansion of the system. However, while $a(t)$ changes by many orders of magnitude, the fireball size increases rather modestly, e.g. from 6 to 8-9 fm at its maximal size, for AuAu collisions at full RHIC energy.

The microscopic scale l is the mean free path for weakly coupled systems (weakly coupled QGP or hadronic gas): in the case of strongly coupled QGP (sQGP) it is just the inverse temperature $l = 1/T$. For AuAu collisions at full RHIC energy l changes from .5 to about 1 fm, from initial to hadronization time. Thus the large parameter $R/l = O(10)$ in the region we use hydrodynamics.

Now let us define two new scales. The first is the *sound horizon*

$$H_s = \int_0^{\tau_f} d\tau c_s(\tau) \quad (2.2)$$

where the integral is taken from the formation to freezeout time. At the freezeout the waves just stop where they are, and the matter is split into independent particles.

It is the same idea as suggested by Sunyaev and Zel'dovich for the Early Universe [17]: the initial perturbation (say higher density at some point) creates a sphere of such radius, at which the density is a bit higher than the average: when galaxies are formed they are correlated with that sphere and thus is observed today in their correlation function. It turns out that in the Big Bang this produces a “standard ruler”, which is today of about $H_s \approx 150$ Mps, observed in galaxy’s distribution and in CMB correlations.

One of the main issues discussed in this paper is whether any manifestation of the sound horizon scale can be observed in the Little Bang. For example, one may think of angular correlations with angles

$$\Delta\phi \approx \frac{2H_s}{R} \quad (2.3)$$

or angular harmonics with $m \sim 1/\Delta\phi$. In the cosmology such angular momentum is $l \sim 200$. Going ahead of ourselves, we will show that in our problem of the Little bang, for AuAu collisions at RHIC, we will deal with $m \sim 3$.

The second scale is not important in cosmology and we would like to call it “the viscous horizon scale” R_v . Its verbal definition is that it separates the wavelengths of the sound which *are and are not* dissipated by the viscosity effects. The smooth fireball and fluctuations are described by

$$T_{\mu\nu} = \tilde{T}_{\mu\nu} + \delta T_{\mu\nu} \quad (2.4)$$

The textbook dispersion law for the sound, including the viscosity term, is

$$\omega = c_s k - \frac{i}{2} \frac{4\eta}{3s} \frac{k^2}{T} \quad (2.5)$$

After that Fourier transform puts it into momentum form, after which one can solve the time dependence using the momentum-dependent dispersion relation as well as the imaginary part induced by viscosity.

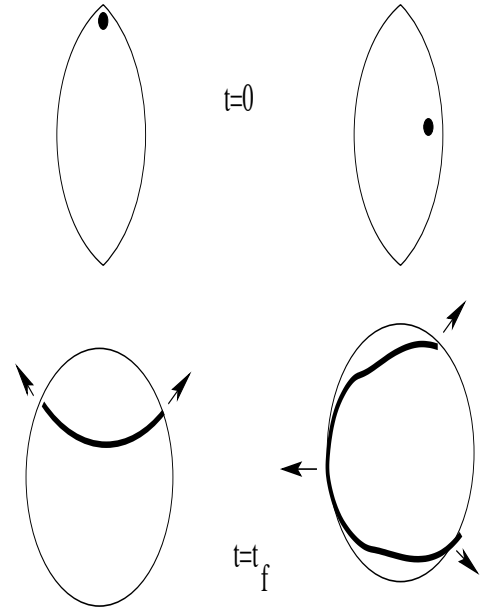


FIG. 4: Two upper picture correspond to initial time $t = 0$: the system has almond shape and contains perturbations (black spots). Two lower pictures show schematically location and diffuseness of the sound fronts at the freezeout time t_f . The arrows indicate the angular direction of the maxima in the angular distributions, 2 and 3 respectively.

(One may add bulk viscosity to this expression as well, but we keep the shear viscosity for now, assuming it is dominant.)

$$\delta T_{\mu\nu}(t) = \exp\left(-\frac{2}{3} \frac{\eta}{s} \frac{k^2 t}{T}\right) \delta T_{\mu\nu}(0) \quad (2.6)$$

The spectrum of the original $t = 0$ perturbations have harmonics of the so called “saturation scale” Q_s , which is for RHIC of the order of $Q_s \sim 1 \text{ GeV}$. Even if one takes the minimal viscosity $\eta/s = 1/4\pi$, by freezeout $t \sim 10 \text{ fm}/c$ this exponent gets very large, damping such fluctuations to an observably small magnitude. Only the harmonics below the new *viscous survival scale* $k < k_v$ would survive, which is determined from the condition that the exponent above is less than 1

$$k_v = \frac{2\pi}{R_v} = \sqrt{\frac{3Ts}{2\tau_f\eta}} \sim 200 \text{ MeV} \quad (2.7)$$

(the number comes from an estimate $\eta/s = 1/4\pi, T \sim 200 \text{ MeV}, \tau_f \sim 10 \text{ fm}/c$).

What it means is that sound perturbation spheres (cylinders, cones etc) would not have the width of the original fluctuations, but they get significantly widened, with the width of the order of $1/k_v$. Note that while the radius of the spheres increases linearly with time $\sim t$, this width increases only as $t^{1/2}$, which means that although

the spheres become more diffuse, they are also relatively sharper and sharper as time goes by.

Let us finish this section by pointing out the main aims of this investigation. By observing propagating sound perturbations one would like to measure the two scales, H_s and $1/k_v$, experimentally, defining two key hydrodynamical parameters – the *speed of sound* and *viscosity*. The way to do so is to change the geometry of the collision (by centrality) and the size of the nucleus (by changing the beam A), and by observing all the harmonics of the flow. The amplitudes of the higher harmonics, dampened by viscosity, if measured, would provide an independent measure of the viscosity.

For central AuAu collisions at RHIC the hierarchy relation between all those four scales is

$$R > H_s > R_v > l \quad (2.8)$$

As some representative numbers let us mention 8, 4, 2, 0.3 fm, respectively. The observation angle of the “peaks” is

$$\Delta\phi \approx 2 \frac{H_s}{R} \quad (2.9)$$

Their angular width is

$$\delta\phi = R_v/R \quad (2.10)$$

so harmonics larger than $m > R/R_v$ can hardly be found, as they are dissipated.

However for mid-central collisions the width (short size) of the “almond” R_x becomes comparable with H_s , and for more peripheral ones $R_x < H_s$. As a result, one expects the sound to traverse the whole fireball and deposit some amount of (entropy) density to its side opposite to the original fluctuation, as it speeded up with the flow. In this case one expects 3-peak events: see Fig.4 for explanation.

So, one of our suggestions to experimentalists is to locate the centrality at which such transition occurs.

III. THE INITIAL STATE FLUCTUATIONS

A. Generalities

Let us start with a comment on what we would call the “initial state”. This term is currently used in at least three different settings:

(i) The wave function of the colliding nuclei, expressed either in terms on the nucleons (their positions in the transverse plane just prior to the collisions) or in terms of partonic degrees of freedom (positions and longitudinal momenta). Another version of it is the “Color Glass Condensate” (CGC) described as an ensemble of classical gauge fields.

(ii) The state just after the (Lorentz contracted) nuclei passed each other. It is either the partonic state, including partons newly produced in a collision; or the so called

GLASMA, in the classical field description.

(iii) The state after approximate equilibration is reached, so that macroscopic (hydrodynamical) description can be started.

It is the last one which we mean in this work, as we would apply hydrodynamics as a tool, translating properties of the initial conditions into the final state observed in the experiment. Therefore our “initial state” should correspond to about one unit of the relaxation time after the actual collision, or numerically at a proper time of the order of 1/2 fm/c. Thus the inhomogeneity of the initial wave functions should be already smoother than at time zero, by this (so far poorly understood) relaxation process.

As we detail below, this state will be described by some “average” or zeroth-order shape of the fireball (depending of course on the impact parameter, the colliding nucleus and the collision energy), plus “fluctuations” characterized in the first order by an ensemble of small perturbations of the average shape described by Fourier coefficients and phases $\{\epsilon_n, \psi_n\}$. Generic expressions would include the zeroth order ensemble-average deformations $\langle \epsilon_n \rangle$ and deviations which have no average but fluctuations $\delta\epsilon_n^2 = \langle \epsilon_n^2 \rangle - \langle \epsilon_n \rangle^2$.

The simplest situation, happening for the second harmonics and sufficiently peripheral collisions, is that the average is much larger than the fluctuations, $\langle \epsilon_2 \rangle \gg \delta\epsilon_2$. If so, one may assume Gaussian form of the fluctuations with the width given by $\delta\epsilon_2$. But the situation is quite different for near-central collisions, for which both terms in ϵ_n come from fluctuations. Their distribution are obviously non-Gaussian because they are all positive by construction. So one must introduce and study their higher powers or cross-correlations (such as $\langle \delta\epsilon_{n_1} \delta\epsilon_{n_2} \delta\epsilon_{n_3} \rangle$ and their generalizations with certain combination of phases (see below). All of those should in principle be provided by the “initial state models”, of which we select Glauber model as the simplest example.

The separation of the initial state fluctuations from all other fluctuations (e.g. fluctuations during the hydrodynamical evolution, hadronization and the freezeout) is possible because of the fundamentally different number of relevant degrees of freedom defining their magnitude. As we will detail in the next section, the so called Glauber fluctuations due to various number of “wounded” (or participant or interacting) nucleons are of the order of

$$\epsilon_n \sim \frac{1}{\sqrt{N_p}} \quad (3.1)$$

where the number of the participant nucleons $N_p \sim O(100)$, being limited from above by the total nucleon number $2A \sim 400$.

Further fluctuations are determined similarly, but with the number of participants N_p substituted by the *much larger* number of partons involved, or the total multiplicity $N_{hadrons} \sim 10^4$ (for RHIC and LHC it is factor 2 different). That is why one may, to certain accuracy,

ignore all later-time fluctuations and assume that observable fluctuations in particle spectra and correlation functions are one-to-one translated from the initial state ensemble. Thus we use hydrodynamical equations as a fully deterministic tool, by itself producing no random numbers at all.

Furthermore, for near-central collisions all $\delta\epsilon_n$ are small, of the order of several percents. So, independently of their possibly complicated distributions and cross correlations, the hydrodynamics applied in linear approximation should be quite reliable tool. Thus hydro equations can be linearized and the linear response coefficients $\delta v_n/\delta\epsilon_n$ calculated. If so, it does not matter what the actual magnitude of the deformation $\delta\epsilon_n$ is. Also the linearized perturbations do not interact with each other.

Although we will focus on those calculations in our next paper, let us note here two things. One simple fact is that while angles ψ_n of the fireball deformations indicate the *maxima* of the distribution (the corners of triangle, square and other polygons), hydro flow goes along their *sides*. Therefore the observed flow angles ξ_n are rotated from the deformation angle as follows

$$\xi_n = \psi_n + \frac{\pi}{n} \quad (3.2)$$

Our second comment is that higher harmonics n are supposed to become oscillatory in time, displaying acoustic sound properties. Naively one may think that at freeze-out this leads to their random phases, and thus those can be ignored. This is however not true, as only the coherent sum of all harmonics with their correct phases will reproduce a propagating sound wave. Finally, if needed, nothing prevents one from solving full nonlinear hydrodynamics (without linearization). In fact it is done by a few groups devoted to “event-to-event hydrodynamics”. If done one can also calculate the nonlinear effects, e.g. appearance of the 4-th harmonics v_4 in spectra coming not from $\delta\epsilon_4$ but from $(\delta\epsilon_2)^2$.

B. Fluctuations in the Glauber model: the amplitudes

Our “Glauber model” is a bit different from that used widely by experimentalists. Both assume that initial state fluctuations originate from the nuclear wave functions. The “usual Glauber” uses randomly placed coordinates of the individual nucleons in the nuclear wave function. However, the nucleons themselves are complicated objects and their interactions are also strongly fluctuating: since there are studies of that we decided to include this source of fluctuations also. This changes numbers a bit, but was found not to be important for any of the qualitative conclusions to be reached.

The nucleon fluctuations we included via the *fluctuating NN cross sections* which are to a certain degree known and studied via diffraction, see [19] for the details and earlier references. Naively, from the well known

fact of a nucleon being made of quite a large number of partons one might conclude that those fluctuations are small, $O(1/N_{partons})$: but this is *not* the case. In our simulation we have assumed the cross section σ_{NN} to be the random Gaussian variable with the variance

$$w_{NN} = \frac{\langle \sigma_{NN}^2 \rangle - \langle \sigma_{NN} \rangle^2}{\langle \sigma_{NN} \rangle^2} \approx 0.25 \quad (3.3)$$

First, like in [12], we simulate a large ensemble of collisions and calculate the magnitude of the ϵ_n for several lowest harmonics (up to 6). Their definition is via the Fourier expansion for a single particle distribution

$$f(\phi) = \frac{1}{2\pi} \left(1 + 2 \sum_n \epsilon_n \cos(n(\phi - \psi_n)) \right) \quad (3.4)$$

where the ϵ_n are the participant anisotropies and the ψ_n are the angles between the x axis and the mayor axis of the participant distribution.

The participant anisotropies are calculated from

$$\epsilon_n = \frac{\sqrt{\langle r^n \cos(n\phi) \rangle^2 + \langle r^n \sin(n\phi) \rangle^2}}{\langle r^n \rangle} \quad (3.5)$$

This expression is calculated in the center of mass of the participant nucleons for each event. Therefore the dipole moment $n = 1$ made out of the average coordinates

$$\langle x \rangle = \langle r \cos(\phi) \rangle = 0, \quad \langle y \rangle = \langle r \sin(\phi) \rangle = 0 \quad (3.6)$$

are zero by definition.

The 2-d shape of the event can in principle be expanded in the double Taylor series in x, y or in double series over moments $r^m \cos(n\phi), r^m \sin(n\phi)$ with integer m, n . An even better definition would be to follow the customary statistical trick and write the distribution as the *exponent* containing the “generating function” of the angular dependence expended in harmonics

$$P = F_1(r) \exp(F_2), F_2 = \sum_{n>0} r^n \epsilon_n \cos(n(\phi - \psi_n)) \quad (3.7)$$

In this way the positivity of the distribution function, as well as inclusion of trivial higher order effects are ensured.

Since the dipole $m = n = 1$ term is zero by construction, we define the first odd deformation ϵ_1, ψ_1 using the term of the expansion $m = 3, n = 1$ which appears together with the triangular deformation $m = n = 3$

$$\epsilon_1 = \frac{\sqrt{\langle r^3 \cos(\phi) \rangle^2 + \langle r^3 \sin(\phi) \rangle^2}}{\langle r^3 \rangle} \quad (3.8)$$

The anisotropies calculated in this way are plotted in figure 5 for $n = 1, 6$. The plot shows that the eccentricity has the largest value for the well known elliptic deformation ϵ_2 and a nonzero value of triangularity ϵ_3 , in agreement with the results reported in [12]. Note that for the near-central collisions $N_{part} > 300$ the elliptic

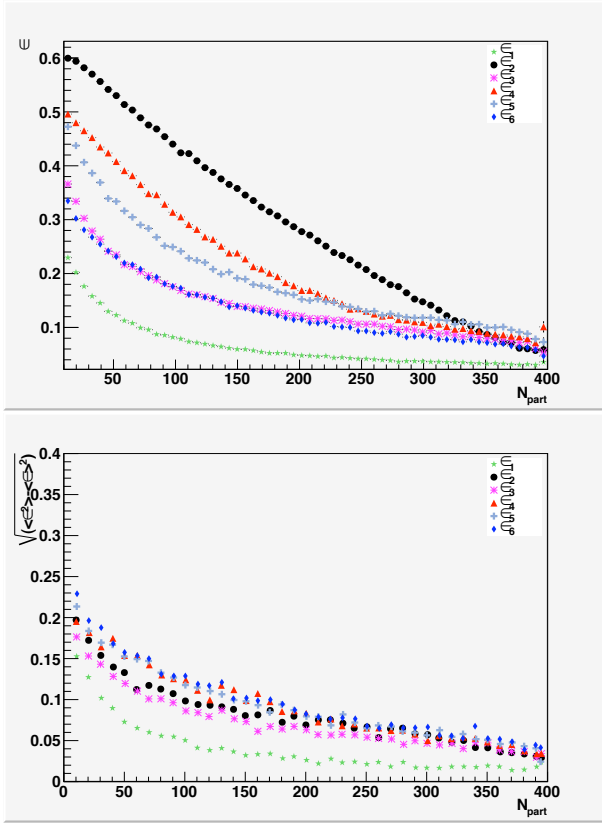


FIG. 5: Average anisotropies (upper plot) and their variations (lower), as a function of centrality expressed via the number of participants N_{part}

deformation is no longer dominant, and it is also due to fluctuations. This conclusion becomes evident as one looks at the lower plot in Fig.5, which shows the variations of these ϵ_n .

One observation coming from these results is that all other deformations (except for ϵ_1 , small because the “true dipole” remains zero) are all comparable, ranging from $O(1/10)$ for central collisions to 0.3 -0.5 for most peripheral ones. While in the central bins these perturbations can be considered small and treated as Gaussian random variables, it is clear that for most peripheral bins (when the number of participants is smaller) the fluctuations are large and thus must be non-Gaussian.

Another consequence is that there is absolutely no ground to single out ϵ_3 : in fact both ϵ_4 and ϵ_5 are larger than ϵ_3 and ϵ_6 is about of the same order as ϵ_3 .

The last point is that their variations (the lower plot) are all comparable to the magnitude. Yet the definition of deformations are such that they are always positive, for each event. This is one more reason that the amplitudes cannot have Gaussian distribution, deviating from it at least for the smallest values.

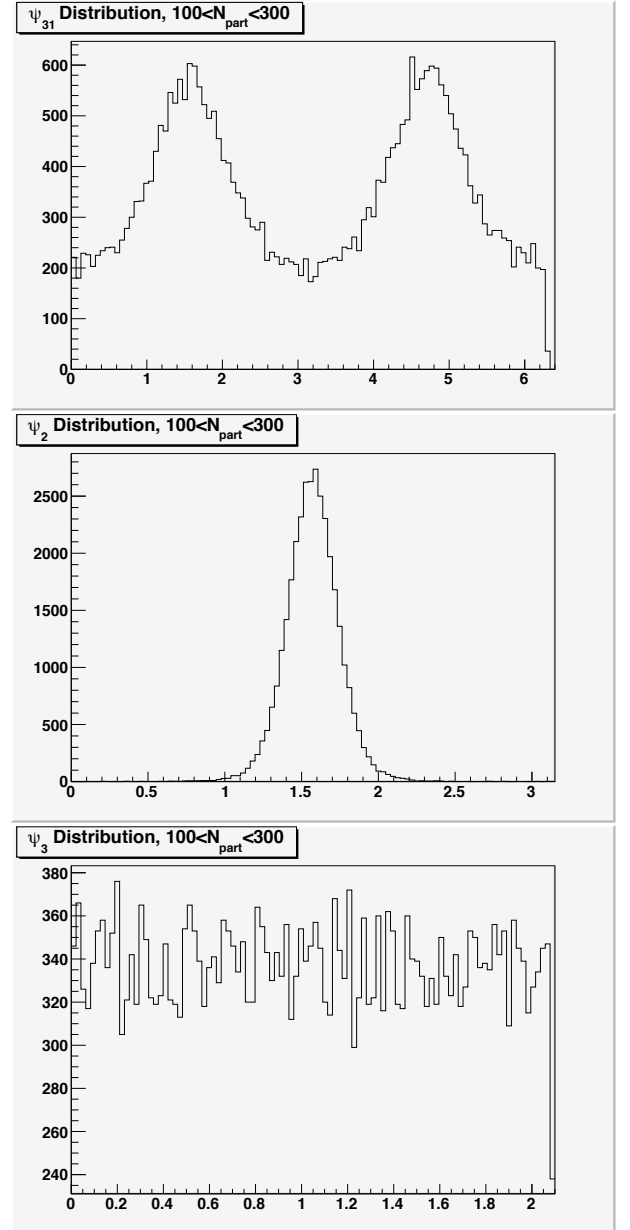


FIG. 6: Distribution of the angles ψ_n for the first three harmonics, the centrality bin used is $100 < N_{part} < 300$

C. Fluctuations in the Glauber model: the angles

The angles ψ_n are defined by:

$$\tan(n\psi_n) = \frac{\langle r^n \sin(n\phi) \rangle}{\langle r^n \cos(n\phi) \rangle} \quad (3.9)$$

and to calculate ψ_1 we use:

$$\tan(\psi_1) = \frac{\langle r^3 \sin(\phi) \rangle}{\langle r^3 \cos(\phi) \rangle} \quad (3.10)$$

Using these expressions we obtain the distribution of the ψ_n 's for the first six harmonics as shown in Figs 6, 7. In

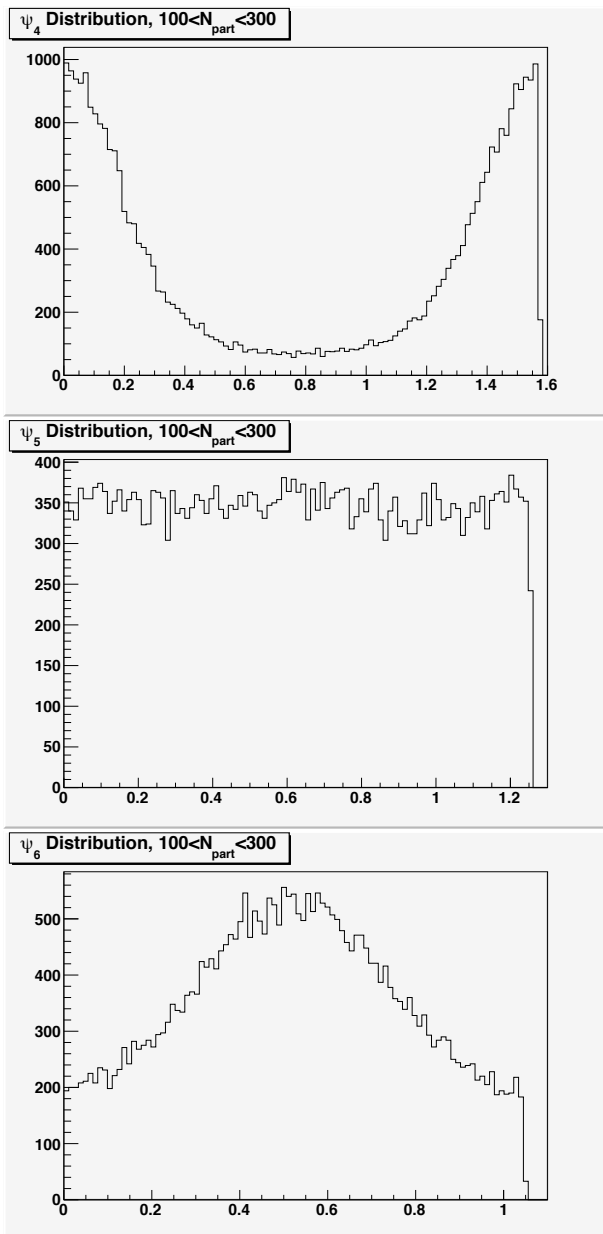


FIG. 7: Distribution of ψ_n for the harmonics 4-6, same centrality

order to better understand the behavior of these angles we will now study their correlation.

(Note that our angle definition is different from the one by Alver-Roland [12]: we do not include extra phase π/n between the flow and deformation directions, see below.)

Let us comment on these distributions, starting from the even ones.

The most obvious one is a distribution of the second (elliptic) harmonic: as seen in Fig.6 the angle ψ_2 is strongly peaked at $\pi/2$, corresponding to an elongation of the system in the y direction, as of course one expects from the overlap “almond” of two nuclei. The distribution of the 4-th angle ψ_4 in Fig.7 shows peaks at angles 0 and $\pi/2$: but since quartic symmetry of the

4-th harmonics it simply means that the maxima of the distribution tend to be aligned with the coordinate axes x and y . The distribution of the 6-th harmonics is different: it is peaked at the angle $\pi/6$. This means that it has no maximum at x direction but rather in y . In conclusion, all even harmonics are strongly correlated with the reaction plane, all of them producing maxima along the y (out-of-plane) direction.

The distribution of the angle ψ_1 is nonzero at all angles, which means it is not very strongly correlated with the reaction plane. It has two maxima, at $\pm y$ directions, to be called “tip” fluctuations. Although the contribution from angles 0, π or x -directions is about twice smaller, it also makes an important contribution: we will call it “waist” fluctuations. Note that while the area of the “waist” is larger than “tips”: and yet its contribution is smaller.

The distribution over ψ_3, ψ_5 in these figures looks completely uncorrelated with the reaction plane. (This fact has also been noticed in [12] and by others.) However, further scrutiny shows that they are in fact well correlated with ψ_1 , see Fig. 8 (in which we included points repeated by periodicity). The distribution can be crudely characterized by some “bumps” plus “stripes” connecting them.

The interpretation of the “bumps” is that all of them correspond to events with additional “hot spot” at the “tips” of the almond. It is a very natural place for maximal fluctuations, for two reasons. First, this is where the participant density in both nuclei is near zero. Second, because of the factor r^3 they have larger weight than all other places.

There are two kinds of “stripes”, with positive and negative slope in Fig. 8. The latter one simply follow from ψ_1 distribution, while the former one is indeed a nontrivial correlation between the angles whose origin we cannot explain. We will continue to discuss its manifestation a bit later. The correlation of ψ_5 with ψ_1 is very similar. The “bumps” at $\psi_5 - \psi_1 \approx 0$ again mean $\pm y$ the direction or the “tips”. The plot has similar “stripes”.

Going a bit ahead of ourselves, let us study the “resonant” combinations of angles, as well as angles and amplitudes. As we explain below, those particular combinations of the amplitudes and phases or two harmonics are

$$\langle \epsilon_{n_1} \epsilon_{n_2} \cos(n_1 \psi_{n_1} - n_2 \psi_{n_2}) \rangle \quad (3.11)$$

especially in the case when n_1, n_2 differ by two units. We have studied two first examples of the kind, with odd harmonics 1,3,5.

One interesting distribution, shown in Fig.9, is that over the \cos term itself, for the particular combination of the 1-3 phases. It consists of two clearly different parts: a very narrow peak near -1 and wide flat distributions between -1 and 1 . This plot demonstrates qualitative feature of the phase distribution which was pointed out above. One explanation of the peak near -1 (the angle combination is π) comes from the fluctuations at the

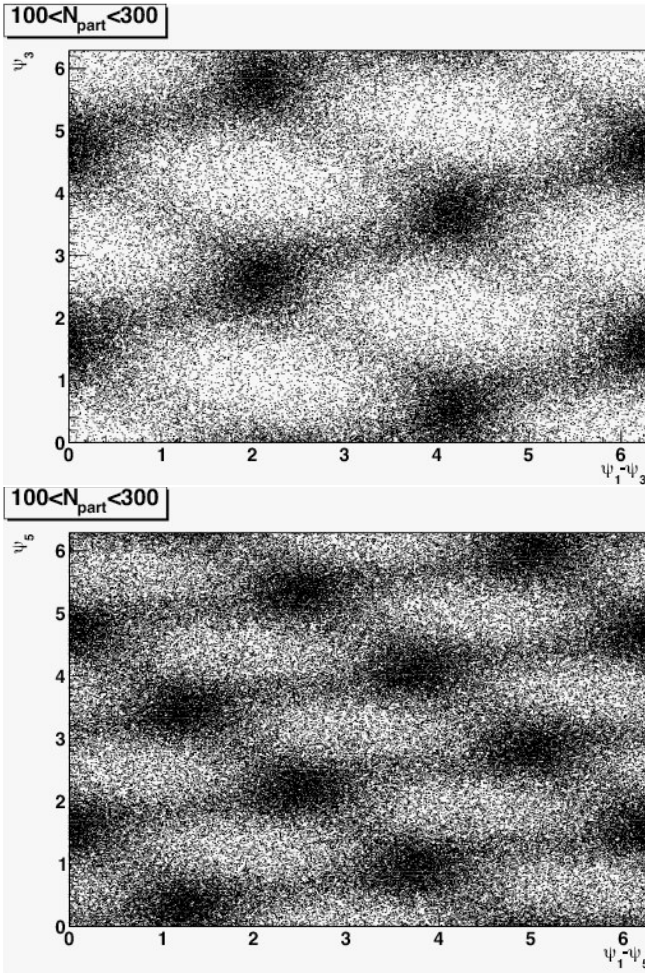


FIG. 8: Scatter plot of the ψ_3 vs $\psi_3 - \psi_1$ (above), and of the ψ_5 vs $\psi_5 - \psi_1$ (below), the same centrality

“tips” of the almond, when both ψ_1 and ψ_3 are strongly correlated close to $\pi/2$. However, the second interesting correlation seen as “positive slope lines” in Fig.8a because for them $\psi_1 - 3\psi_3 = \pi$ as well. A similar situation happens for other odd harmonics.

The average value of the combinations (3.11) for 1-3 and 3-5 harmonics as a function of centrality are shown in Fig.10. All values are negative, as the sign is dominated by a peak in \cos near -1: the other component more or less averages out. We thus conclude that experimental measurements of the amplitude of such correlations, with the magnitude and the sign, will be especially sensitive to the “almond tip” fluctuations.

Summarizing the observed pattern: we have found that all odd angles are well correlated with each other, forming the “stripes” and “bumps” shown in Fig.8. The fluctuations and correlations seem to be stronger from the “tips” of the almond.

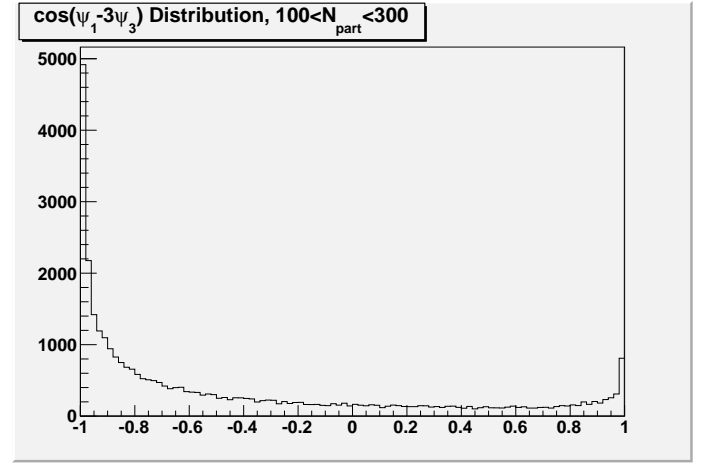


FIG. 9: Scatter plot of $\cos(3\psi_3 - \psi_1)$

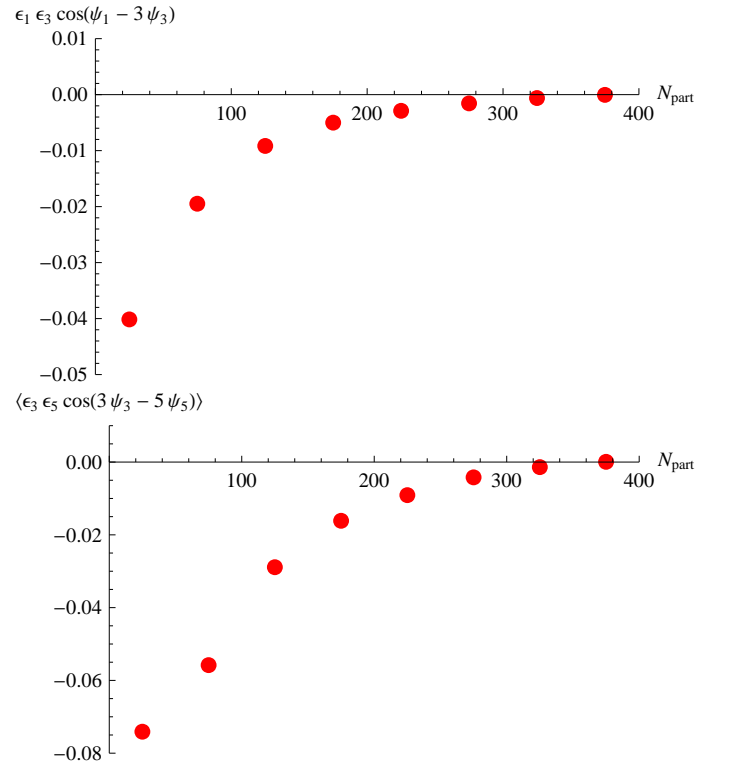


FIG. 10: Correlators $\langle \epsilon_1 \epsilon_3 \cos(\psi_1 - 3\psi_3) \rangle$ (top) and $\langle \epsilon_3 \epsilon_5 \cos(3\psi_3 - 5\psi_5) \rangle$ (bottom) as a function of the number of participants. The error bars are omitted since they are smaller than the dots.

D. Comments on other initial state fluctuations

So far the only source of fluctuations included has been (i) the coordinate part of the nuclear wave function prescribing the nucleon positions in the transverse plane, and (ii) the event-by-event fluctuation of the NN cross section. We found that the former effect dominates and the latter only provides small corrections.

While other sources of fluctuations clearly are subject for future studies, we still provide some comments on

those.

One important type of “initial state” fluctuations is of course hard parton scattering events, resulting in jet production. The rate of those very strongly depend on the exact definition of the cutoff beyond which the momentum transfer involved is characterized as “hard”. There are vastly different opinions on where this boundary is theoretically, and experimentally it depends on whether such events are triggered by single large- p_t hadron or by some jet-finding algorithms. Jet production and quenching is of course of high interest, but those should be studied only in small fraction of all collisions selected by separate triggers. For global fluctuations those can safely be ignored, as the probability of “hard” events is smaller than that of the fluctuations we study.

In the Glauber approach that we used, the local density of produced matter is assumed to be simply proportional to the local density of all participant nucleons, $N_p(A_1) + N_p(A_2)$. However, when this density is high enough, it has been argued that the so called “saturation” phenomenon should take place, because of parton absorption processes in the wave function. Other expressions for local matter density have been proposed, e.g. $\sim \min(N_p) \ln(\max(N_p)/\min(N_p))$ by Kharzeev, Nardi, Levin, where min and max refer to the smaller and the larger of the two. Those are typically the amplitudes of the fluctuations.

A well known approach to their description is the so called “glasma”, which calculates those color fields from random color charges of the leading (larger x) partons of the two colliding nuclei. Asymptotically (in a very large nuclei or at very high energy) McLerran and Venugopalan [24] argued that at a particular location in the transverse plane the color charges of partons must be uncorrelated because they come from different nucleons. Therefore it is usually assumed that their color charges fluctuate as random variables. If so, the resulting fluctuations are small, as the total number of partons is very large.

Application of such ideas usually keep the average values of those, such as $A^{1/3}$ or so. The point of our comment is a warning, that such simplified ideas cannot be used for determining the fluctuations. It has been known for a long time, that while at very small x the partons, mostly gluons, become numerous, they are still very tightly correlated in the transverse plane. The size of the “gluonic spot” in the nucleon has been known for long time from diffractive form factors, and in more recent form, from HERA photon diffraction into J/ψ . This spot is small, and therefore bright. As documented e.g. in [21] (their Fig.23), the gluon density at the center of the nucleon is about as high as in the center of Ca_{40} . Therefore, large number of gluons does not yet imply that all of them merge in the transverse plane into more or less homogeneous distribution: the positions of the incoming nucleons still remain the dominant source of the initial state fluctuations.

As the “gluonic spots” from single nucleons remain the source of initial state fluctuations, one may ask if numer-

ous partons coming from it may be correlated in their quantum numbers, forming specific large-amplitude color fields. One particular kind of such fields got special attention: those are the topologically nontrivial gluonic configurations called *the QCD sphalerons* [25]. They are gluonic field configurations which originate from excitations of the topologically nontrivial vacuum fields (instantons). While they rapidly explode into multiple gluon state, they strongly violate CP and chiral symmetries locally, producing in particular $\pm 2N_f$ (≈ 6) units of chirality per sphaleron. As pointed out in [26], such strong local CP violation induces special event-by-event fluctuations in the CP-odd observables, e.g. they should induce charge asymmetry along the magnetic field. Clearly those should be looked at in special studies.

Another coherent color field configurations which deserve to be specially studied are (colorelectric) flux tubes. In pp collisions the view that a field created by longitudinally separated charges makes a flux tube is a consequence of confinement, and thus must happen in vacuum. Many popular event generators are based on the Lund model, cleverly parameterizing flux tube production and decay. In AA low energy collisions many flux tubes are produced, and their possible fluctuations into the so called “color ropes” has been studied, initiated by the paper of [22]. If two elementary color charges can be combined, they may either cancel each other or produce higher representations of the SU(3) group, in which case the rope energy (and entropy, after its decay) is proportional to its flux *squared*. Further applications of these ideas for strangeness production in AA collisions can be found in [23].

Studies of the flux tubes lay dormant till recent discovery of the so called “hard ridge” [28] by STAR collaboration at RHIC. One possible origin of it [13] is hydro-carried longitudinal flux tube, created at the hard collision point. This explanation may work provided the flux tubes survive as such till freezeout: as was argued in that paper this indeed should happen at the periphery of the fireball, where matter is not far from the deconfinement transition, forming a kind of “dual corona” of the QGP fireball similar to Solar corona full of flux tubes.

IV. WHICH CORRELATIONS SHOULD ONE STUDY?

A. Central collisions: two versus many-particle correlators

Suppose a given event has certain (2-dimensional) distribution over transverse momenta of the secondaries. This distribution can be decomposed into Taylor series of the momenta p_x, p_y or into the angular harmonics

$$\frac{dN}{dp_t^2 d\phi} = f(p_t) \left[1 + \sum_{n>0} (2a_n \cos(n\phi) + 2b_n \sin(n\phi)) \right] \\ a_n = \langle \cos(n\phi) \rangle, b_n = \langle \sin(n\phi) \rangle \quad (4.1)$$

(Note that instead of the square bracket one can also use the exponent of the sum, which will include trivial higher order effects and enforces positivity of the distribution: but we would assume all v_n to be very small, for simplicity.) Instead of using the a, b pair, one may also introduce the modula and the phases writing it as $2v_n \cos[n(\phi - \xi_n)]$ with positive v_n . In order to simplify subsequent formulae, we however prefer to write it using the complex exponent

$$\frac{dN}{dp_t^2 d\phi} = f(p_t) \left(\sum_n v_n e^{in\phi - in\xi_n} \right) \quad (4.2)$$

where the sum goes over all integer n , positive and negative, with $v_0 = 1$ and $v_{-n} = v_n$.

The sum of the harmonics presumably describe some interesting shapes. For example, those can be the sound circles from point perturbations, leading to two-maxima shapes mentioned in the Introduction. Unfortunately, there are multiple perturbations in any event, and the fluctuations we discuss are so small that they can only be studied by finding statistically significant small correlations, using large ($\sim 10^9$) sample of available events.

Discussing how to do it let us, for simplicity, first discuss a subset of exactly central collisions, with zero (negligibly small) impact parameter and thus exact azimuthal symmetry of the event ensemble. Individual events would have of course some fluctuations pointing to some directions ξ_n , but their absolute directions are arbitrary, not defined relative to anything physical. Whatever fluctuation happens, it is clear that its copy rotated by any angle should also exist and have the same probability. Say, if the elementary perturbation is local (delta-function-like in the transverse plane): its angular position in the transverse plane is the only meaningful azimuthal orientation. Let us express it as

$$\xi_n = \xi_p + \tilde{\xi}_n \quad (4.3)$$

where the tilde indicates the angle *relative* to the perturbation. Obviously ξ_p is a random variable, changing from event to event, and thus should be averaged out. It turns out that the way the correlations work out is quite different for (i) the two-body and (ii) the many-body (three or more) correlation functions. Indeed, in order to get the two-body correlation function one has to take a the square of the single-body distribution (4.2)

$$\sum_{n_1, n_2} v_{n_1} v_{n_2} \exp[in_1 \phi_1 + in_2 \phi_2 - in_1 \tilde{\xi}_{n_1} - in_2 \tilde{\xi}_{n_2} - i(n_1 + n_2) \xi_p]$$

and average it over ξ_p randomly distributed over the circle. As a result only terms satisfying

$$n_1 + n_2 = 0 \quad (4.4)$$

survive, which has three important consequences. First, a double sum collapses into a single sum with the squared

amplitude ϵ_n^2 . Second, it becomes a function of the angular *difference* $\Delta\phi = \phi_1 - \phi_2$, as expected. And, last but not least, all the *phases* ξ_n *disappear*.

This facts are of course well known, usually expressed as harmonics of the 2-body correlator

$$C_2(\Delta\phi) = \langle \frac{d^2 N}{d\phi_1 d\phi_2} \rangle_{|\xi_p} \quad (4.5)$$

$$c_{n\Delta} = \frac{\int d(\Delta\phi) C_2(\Delta\phi) \cos(n\Delta\phi)}{\int d(\Delta\phi) C_2} = \langle v_n^2 \rangle \quad (4.6)$$

So, the two-body correlator provides *squared* amplitudes of the original harmonics, *averaged* over the events. This is e.g. how Alver and Roland [12] and others have obtained their estimates for the “triangular” flow.

However, the situation is different for manybody (three or more) correlation functions. Indeed, if the single-body distribution (4.2) is cubed (or raised into higher power), one finds a *triple* sum in which random perturbation direction appears as $\exp[i(n_1 + n_2 + n_3)\psi_p]$. Averaging over it leads now to the condition

$$n_1 + n_2 + n_3 = 0 \quad (4.7)$$

Eliminating e.g. n_3 one finds the double sum

$$\sum_{n_1, n_2} v_{n_1} v_{n_2} v_{n_1+n_2} \exp\{i[n_1(\phi_1 - \phi_3) + n_2(\phi_2 - \phi_3) - n_1(\tilde{\xi}_{n_1} - \tilde{\xi}_{n_1+n_2}) - n_2(\tilde{\xi}_{n_1} - \tilde{\xi}_{n_1+n_2})]\}$$

in which the relative orientations of the different harmonics are still present. Taking corresponding moments over the observed angle differences $\phi_1 - \phi_3, \phi_2 - \phi_3$ one can single out the corresponding terms, and thus measure the combination

$$\langle v_{n_1} v_{n_2} v_{n_3} \cos(n_1 \xi_{n_1} + n_2 \xi_{n_2} + n_3 \xi_{n_3}) \rangle \quad (4.8)$$

in which three integers are subject to the “resonance” condition (4.7). Therefore, since there are many moments of various many-body correlators, much more complete data analysis is possible, ultimately allowing to observe also the *phases* of the harmonics. The price to pay is that the smallness of all harmonics now appear in the third (or higher) power, so it is more difficult to get its values out of the statistical noise.

Let us now briefly discuss how comparison to theory should be done, assuming such averages are experimentally measured. There are two steps to be done. First, using linear response hydro one can approximate it as

$$\begin{aligned} & \langle v_{n_1} v_{n_2} v_{n_3} \cos(n_1 \xi_{n_1} + n_2 \xi_{n_2} + n_3 \xi_{n_3}) \rangle \\ &= \left(\frac{v_{n_1}}{\epsilon_{n_1}} \right) \left(\frac{v_{n_2}}{\epsilon_{n_2}} \right) \left(\frac{v_{n_3}}{\epsilon_{n_3}} \right) \langle \epsilon_{n_1} \epsilon_{n_2} \epsilon_{n_3} \cos(n_1 \xi_{n_1} + n_2 \xi_{n_2} + n_3 \xi_{n_3}) \rangle \end{aligned} \quad (4.9)$$

Second, one should change from the flow angles to deformation angles using (3.2). Note that in each three terms

n_i in numerator and denominator cancel, leaving only 3π or simply the sign change

$$\begin{aligned} \cos(n_1\xi_{n_1} + n_2\xi_{n_2} + n_3\xi_{n_3}) = \\ -\cos(n_1\psi_{n_1} + n_2\psi_{n_2} + n_3\psi_{n_3}) \end{aligned} \quad (4.10)$$

The resulting correlation of the amplitudes and orientations of the initial state fluctuations can be calculated from initial state model, as we have done above for the first three harmonics resonance $1 + 2 = 3$ as well as $3 + 2 = 5$.

In fact it is not necessary to take angular moments of the correlation functions. Instead of lengthy but obvious general expressions, let us give a simple but instructive example. We are interested in telling the difference between the two-prong events predicted by sound propagation and three-symmetric case of purely triangular flow.

Consider the two simplest shapes depicted in Fig.11. The upper figure has (green) curve which shows the two-peak distribution, and the lower figure has (red) three-peak one. If it is projected into angular harmonics, and the (viscous) filter kills all but the first three, one gets their approximation by the other lines. The 1-st and 3-ed harmonics are correlated because of absent third peak.

Defining the 3-particle correlator as for the 2 particle one, with the averaging over the perturbation angle

$$C_3(\phi_1, \phi_2, \phi_3) = \langle \frac{d^2N}{d\phi_1 d\phi_2 d\phi_3} \rangle_{\psi_p} \quad (4.11)$$

One finds it to be a function of two angles, say $\phi_1 - \phi_3, \phi_2 - \phi_3$. The corresponding plots for two examples in question, the 2-peak and 3-peak distributions, is shown in Fig.12. Comparing two plots one can see how different they are. In experiment, which will produce a mixture of the two, one would see that intermediate pattern, and hopefully determine the presence and probability of both type of events.

B. Mid-central collisions and the two-body correlators relative to the event plane

Nonzero impact parameter violates axial symmetry and creates “directed flows”, e.g, the famous elliptic flow with nonzero $\langle v_2 \rangle \neq 0$. By mid-central collisions we mean a centrality region in which $\langle \epsilon_2 \rangle$ is large itself, and is also large compared to its fluctuations (recall that it is not so for central and very peripheral collisions). For example, ϵ_2 is 0.5 (0.3) for $N_p = 100$ (200) participants, with $\delta\epsilon_2 \approx 0.1$. Furthermore, as seen in Fig.6b, its angle ψ_2 is very much directed at $\pm\pi/2$ (the tips of the almond) and therefore (using (3.2 for $n = 2$) the flow angle is peaked “in-plane”, $\xi_2 = 0, \pi$, as indeed observed.

If so, for one of the harmonics being the second e.g. $n_3 = \pm 2$ one can approximate a product of three deformations as follows

$$\begin{aligned} & \langle v_{n_1} v_{n_2} v_{n_3} \cos(n_1\xi_{n_1} + n_2\xi_{n_2} + n_3\xi_{n_3}) \rangle \\ & \approx \left(\frac{v_{n_1}}{\epsilon_{n_1}} \right) \left(\frac{v_{n_2}}{\epsilon_{n_2}} \right) \left(\frac{v_2}{\epsilon_2} \right) \langle \epsilon_2 \rangle < \epsilon_{n_1} \epsilon_{n_2} \cos(n_1\xi_{n_1} + n_2\xi_{n_2}) \rangle \end{aligned} \quad (4.12)$$

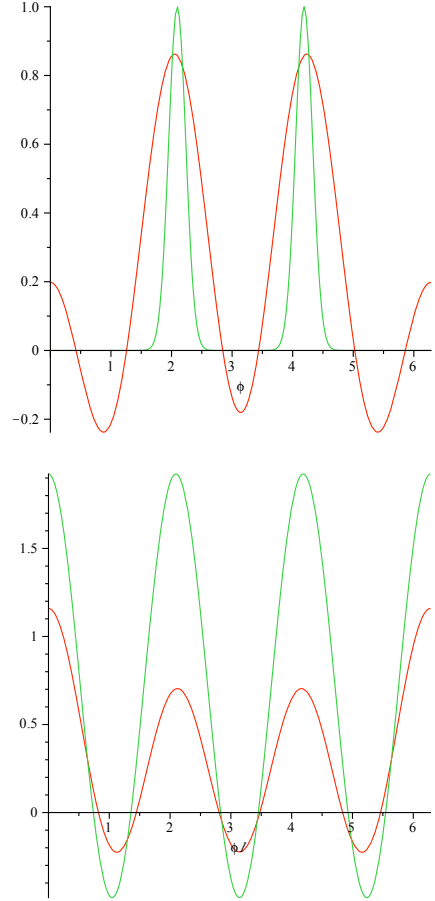


FIG. 11: (upper) Two-peaked (green) curve shows the simplest model of angular distribution to be discussed. The (red) oscillating curve is its version after the filter removes all but harmonics with $n > 3$.

(lower) The lower (red) curve shows the dependence of the 2-particle correlation function on $\phi_1 - \phi_2$ for the filtered 2-peak distribution. The higher (green) curve is shown for comparison, it corresponds to another model distribution with three identical peak shifted by 120° .

by factoring out large and non-fluctuating $\langle \epsilon_2 \rangle$ from two other harmonics which are small and fluctuating. Note that the resonance condition now means $n_1 \pm 2 = n_2$, and that by putting $\xi_2 = 0$ we have selected a frame in which the (experimentally determined reaction plane) is the x axis.

Basically the lesson here is that for mid-central collisions the “reaction plane” plays the same role as the third body, so we are reduced to two small fluctuating and correlated harmonics. The simplest nontrivial example of resonance condition of the kind is $3-1-2=0$ (recently studied by Teaney and Yan [27]), while the next is $5-3-2=0$.

We had already calculated the combinations of two fluctuating harmonics with the appropriate cosines above, for these two cases, in the Glauber model. They

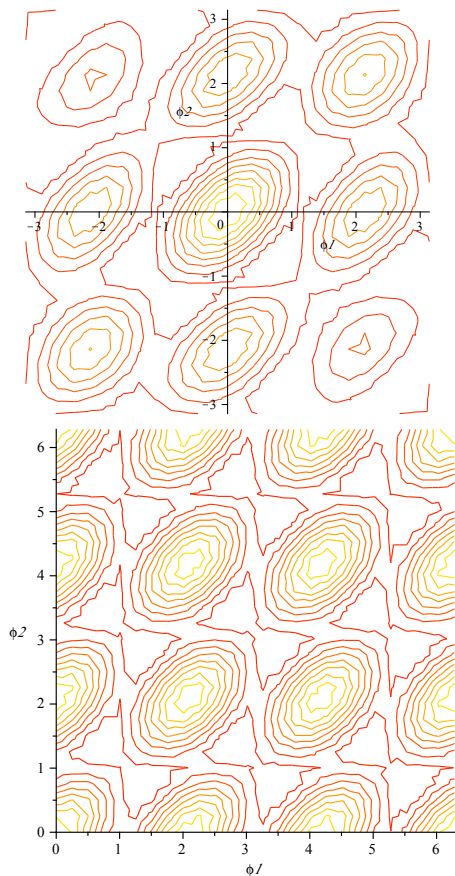


FIG. 12: The contour plot of the three particle correlators as a function of $\phi_1 - \phi_3$ and $\phi_2 - \phi_3$, for 2-peak distribution (upper figure) and 3-peak one (lower). As one can see, their shapes are quite different.

are by no means small: for example for the centrality class with 100 participants they are

$$\langle \epsilon_1 \epsilon_3 \cos(3\psi_3 - \psi_1) \rangle \sim -0.015 \quad (4.13)$$

$$\langle \epsilon_3 \epsilon_5 \cos(3\psi_3 - 5\psi_5) \rangle \sim -0.05 \quad (4.14)$$

and therefore we expect it to be observable, with about as large statistics as needed for the usual quadratic fluctuations.

V. SUMMARY

In this work we have (i) discussed the setting, identifying the main scales of the problem. Then (ii) we studied in detail the initial state fluctuations originating from nucleon positions, emphasizing existence of the nontrivial phase relations between different harmonics. Finally, (iii) we discussed correlations functions of 2 and many hadrons, pointed out the principle difference between them, with the latter allowing to experimentally measure the relative phases of these harmonics.

Let us now recapitulate the lessons from this study in a bit more detail.

Unfortunately, the perturbations we speak about are too small to be measured directly, on event-by-event basis, and should be instead reconstructed from the statistically obtained correlation functions. One good thing coming from it is that multiple but independent fluctuations from local perturbations in a single event and in the ensemble are treated in one and the same correlation function, while all trivial effects are statistically subtracted. Traditionally the initial state perturbations and final state corrections to collective flow are considered in a form of their angular harmonics, which we call ϵ_n and v_n respectively. Their ratios v_n/ϵ_n can be calculated by the linearized hydrodynamics: the details of that is the subject of our next paper.

“Harmonic flows” have been so far treated as independent or incoherent fluctuations, added in quadratures. We however pointed out that it is only true for say central collisions and two-particle correlations. For central collisions many (3 and more) particle correlations already include phases of the deformations ψ_n , as do two-body correlations relative to event plane for mid-central collisions. Many different moments related to “resonances” between 3 integers can be measured, providing experimental opportunity to find out how the perturbations are correlated and eventually to refine models of the initial state. The magnitude of such terms, as we have shown for Glauber model, is no smaller than for the terms already studied.

Coherence in phases of the deformations imply the interferences between the harmonics of the flow. Only adding them together one can follow how small “hot” (or “cold”) spots created by quantum fluctuations of interacting nucleons propagate away from the point of origin. Only in this way one can understand the role played by the “hydrodynamical causality”, insisting that large part of the fireball should remain completely unaffected by the perturbation since the signal cannot possibly reach it before the freezeout. Only a complete Green function, collecting all hydro harmonics, will describe correct shapes of propagating waves, such as e.g. (i) the “cylinders” from the initial state fluctuations, or (ii) the “Mach cones” from quenched jets.

There are two basic scales defining those perturbations, the sound horizon H_s (2.2) and the “the viscous horizon scale” R_v (2.7). The former gives the size of the perturbation, stemming from a local perturbation, the second its width. We have for example argued that by changing the centrality of the collisions, one can change the relation between the (smaller) fireball size and the sound horizon: this should dramatically change the shape of the event (see Fig.4 for explanation). It is an important objective of the experimental heavy ion program in general to measure these two scales, extracting experimental values of the speed of sound and viscosity. It can be done for example by changing centrality and observing the change of shape of the underlying event.

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